

Another Alternative to the Higgs Mechanism*

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The Higgs mechanism is designed to generate mass for massless particles. The mass comes from the interaction of observed particles with an external field – the Higgs field. In the past, several alternatives to the Higgs mechanism for mass generation have been proposed to avoid the postulation of the Higgs field. This article proposes yet another one. This alternative is distinctly different from the others because it considers mass generation through *internal* interactions of a particle rather than interactions with external fields. This requires particles to have an internal structure beyond intrinsic spin. A complete field theory of such composite particles is seen to be possible. Of course, if Higgs bosons are observed by experiment, there will be no need for any alternatives. On the other hand, if experiment fails to detect Higgs bosons, such alternate mechanisms for particle mass generation would be very useful.

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I. INTRODUCTION

The elegance of particle physics theory comes from the underlying symmetry groups. Larger the number of particles that fall under one group, the better. Differences in masses of particles can ruin such symmetry. Hence, the Higgs mechanism is found to be very useful. It can generate mass dynamically, thus allowing particles to have zero rest mass in their interaction-free equations of motion. However, the Higgs mechanism necessitates the existence of the Higgs bosons which have not been experimentally detected so far. Hence, mass generation without the postulation of the Higgs field is an interesting possibility. Several such mechanisms have already been proposed[1, 2, 3, 4]. The mechanism proposed here is different from the others. It generates mass through *internal* interactions rather than external interactions with other fields. This requires that internal structure be ascribed to traditional structureless particles. Hence, a field theory of structured (or extended) objects needs to be developed. String theories deal with such extended objects. However, strings having infinite degrees of freedom complicates matters. The theory discussed here considers fields of extended objects with finite numbers of internal degrees of freedom. To maintain relativistic covariance (both classical and quantum) in the treatment of internal interactions of such extended objects one uses methods developed in relativistic hamiltonian constraint dynamics[6, 7, 8, 9, 10, 11, 12, 13]. These methods have been used successfully over many years in models for particle bound states[14, 15, 16].

II. CLASSICAL EXTENDED OBJECTS: REST-MASS AND EFFECTIVE MASS GENERATION

Classically, the mass-shell condition for a free particle is as follows.

$$p^2 + m^2 = 0, \quad (1)$$

where p is the four-momentum of the particle and m its rest-mass. This is equivalent to stating

$$p^{\parallel} = m, \quad (2)$$

where, for any four-vector v , the component parallel to the momentum (zeroth component in the center-of-mass (CM) frame) is given by

$$v^{\parallel} = -v \cdot \hat{p}, \quad \hat{p} = p / \sqrt{-p^2} \quad (3)$$

So, p^{\parallel} is the center-of-mass energy as well as the rest-mass of the particle. Hence, one may make the following rather trivial observation.

Rest-mass is the energy in the CM frame.

However, the same statement is not that trivial when applied to composite objects with interacting components. Consider a point particle with another point object attached to it by some confining force. We shall call the point particle the *vertex* (or the *bare particle*) and the attached point object the *satellite*. Let the position and momentum four-vectors for the vertex be q_0 and p_0 respectively and those for the satellite be q_1 and p_1 respectively. As we are interested in a theory of massless bare particles, we need to focus on the rest-mass of the vertex. The nature of this rest-mass is complicated by the fact that the vertex is bound to the satellite by a confining force. The rest-mass of a free particle is its energy at rest. But the vertex, by itself, is never free. Hence, for the purpose of interacting objects like the vertex, the definition of rest-mass needs to be generalized to

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the above statement which happens to be trivial for free particles – rest-mass is the energy in the CM frame.

For the vertex, its own CM frame is not inertial. So we use the CM frame of the whole composite of vertex and satellite. The total momentum of the composite is

$$P = p_0 + p_1. \quad (4)$$

So the energy of the vertex in the CM frame is denoted by the component of p_0 parallel to the total momentum P :

$$p_0^{\parallel} = -p_0 \cdot \hat{P}, \quad \hat{P} = P/\sqrt{-P^2}. \quad (5)$$

Hence, the rest-mass for the vertex is as follows.

$$p_0^{\parallel} = m. \quad (6)$$

This is the equivalent of the mass-shell condition for free particles (equation 2).

Now, for the sake of symmetry, if the rest-mass of the vertex (or the bare particle) were to be zero, it would have the following mass-shell condition.

$$p_0^{\parallel} = 0. \quad (7)$$

This can be rewritten (using equations 4 and 5) as

$$P^2 + (p_1^{\parallel})^2 = 0, \quad (8)$$

where p_1^{\parallel} is the component of p_1 along P . Naturally, if M were the *effective mass* of the whole composite, then

$$P^2 + M^2 = 0. \quad (9)$$

Hence, we identify the dynamically generated effective mass as

$$M \equiv p_1^{\parallel}. \quad (10)$$

Clearly, this is generated by internal dynamics and can be non-zero while the rest-mass of the vertex is zero. M can be seen to be a constant of motion that depends on initial conditions and the nature of the interaction between vertex and satellite. To see this, one needs to find the relationship of p_1^{\parallel} to the interaction of the vertex and satellite.

Equation 7 is the mass-shell condition for the zero rest-mass vertex. Equivalently (equation 9), it is also the effective mass-shell condition for the composite. But the composite is made up of two point objects and hence, it must have two independent mass-shell conditions as required by relativistic hamiltonian constraint dynamics[6, 7, 8, 9, 10, 11, 12, 13]. The second mass-shell condition is imposed on p_1 , the momentum of the satellite as follows.

$$p_1^2 + m_1^2 = 0. \quad (11)$$

If m_1 were just a constant, it would be the mass of the satellite and require that the satellite be a free particle.

But the satellite is not a free particle. So, m_1 cannot be a constant. Requiring that m_1 be a function of the position of the satellite relative to the vertex, effectively introduces the interaction between the vertex and the satellite. The relative coordinate between the vertex and the satellite is

$$\xi = q_1 - q_0. \quad (12)$$

So,

$$m_1 \equiv m_1(\xi). \quad (13)$$

This also shows that a confined particle (the satellite) can be viewed as something with a variable mass – a mass that increases with distance from the vertex.

Classical hamiltonian constraint dynamics requires that the two mass-shell conditions given by equations 9 and 11 satisfy further conditions for consistency. These conditions can be specified many different ways. However, for a smooth transition from classical to quantum, the following Poisson bracket condition is preferred[6, 9, 10].

$$\{D_0, D_1\} = 0, \quad (14)$$

where,

$$D_0 = P^2 + M^2, \quad D_1 = p_1^2 + m_1^2. \quad (15)$$

It is to be noted that although D_0 and D_1 individually vanish due to the mass-shell conditions, their Poisson bracket can still be non-zero. Hence, equation 14 is a condition independent of the mass-shell conditions. It can be seen that this new condition is satisfied if the function $m_1(\xi)$ is restricted to be a function of ξ^{\perp} alone[6, 17] where, for an arbitrary four-vector v , we define

$$v^{\perp} \equiv v \cdot P^{\perp}, \quad P^{\perp} \equiv \eta + \hat{P}\hat{P}. \quad (16)$$

Here η is the Minkowski metric and P^{\perp} is the projection operator that projects orthogonal to P . Hence,

$$m_1 \equiv m_1(\xi^{\perp}). \quad (17)$$

This states that m_1 must be a function of only the spatial components of ξ in the CM frame.

It can now be noticed that $\sqrt{-P^2}$ is the energy in the CM frame and that

$$\{M, \sqrt{-P^2}\} = \{p_1^{\parallel}, \sqrt{-P^2}\} = 0. \quad (18)$$

Hence, M is a conserved quantity and it can be treated as the dynamically generated effective mass of the composite as indicated by equation 9. As this effective mass is dynamically generated, it will depend on initial conditions. Classically, M can acquire a continuum of values depending on initial conditions. However, in a quantized model it can be restricted to certain discrete values. The following sections deal with the quantization of this composite particle model.

III. FIRST QUANTIZATION OF EXTENDED OBJECTS (COMPOSITE PARTICLES)

To first quantize the composite object described above, a convenient set of phase space variables needs to be identified. The most obvious set is the following.

$$S_{p0} = \{p_0, p_1, q_0, q_1\}. \quad (19)$$

Quantization amounts to converting the Poisson bracket relations of this space to commutator bracket relations. This gives the following non-zero commutator brackets.

$$[q_0, p_0] = i\eta, \quad [q_1, p_1] = i\eta. \quad (20)$$

All other commutators are zero. A canonical transformation of S_{p0} to accommodate the translation invariant ξ is useful. This produces the following phase space variables.

$$S_{p1} = \{P, \pi, Q, \xi\}, \quad (21)$$

where

$$P = p_0 + p_1, \quad \pi = p_1, \quad Q = q_0, \quad \xi = q_1 - q_0. \quad (22)$$

The non-zero commutators of S_{p1} are as follows.

$$[Q, P] = i\eta, \quad [\xi, \pi] = i\eta. \quad (23)$$

A less trivial transformation is to the following set.

$$S_p = \{P, \pi^{\parallel}, \pi^{\perp}, x, \xi^{\parallel}, \xi^{\perp}\}, \quad (24)$$

where the components of π and ξ parallel and orthogonal to P (CM components) are used. However, in this set Q cannot be used any more. This is because the CM components of π and ξ depend on P and hence their commutators with Q do not vanish. However, it can be proved[18] that there exists an x such that the only non-zero commutators of S_p are the following.

$$[x, P] = i\eta, \quad [\xi^{\parallel}, \pi^{\parallel}] = -i, \quad [\xi^{\perp}, \pi^{\perp}] = iP^{\perp}. \quad (25)$$

As long as the existence of x is proven, its explicit dependence on the variables of S_{p1} is not necessary for the discussion of a quantum theory. Due to the commutation relations, x behaves as the position of the composite particle.

The commutation relations of S_p provide the following maximal set of mutually commuting variables.

$$S_L = \{x, \xi^{\parallel}, \xi^{\perp}\}. \quad (26)$$

Hence, the first quantized wavefunction of the system can be written as a function on S_L .

$$\psi = \psi(x, \xi^{\parallel}, \xi^{\perp}). \quad (27)$$

The commutation conditions of equation 25 lead to the following differential operator representation of the momenta.

$$i(-\pi^{\parallel}, \pi^{\perp}) = \partial_{s\alpha} \equiv \left(\frac{\partial}{\partial \xi^{\parallel}}, \nabla \right), \quad (28)$$

and

$$iP_{\alpha} = \partial_{\alpha} \equiv \frac{\partial}{\partial x^{\alpha}}, \quad (29)$$

where α is the four-vector index and the subscript s identifies the derivatives with respect to the satellite relative coordinates ξ . The four-vector component notation (\dots, \dots) gives the zeroth component as the first argument and the three-vector components as the second argument. The three-vector operator ∇ is defined to be the gradient in the three-vector space of ξ^{\perp} which represents the spatial components of ξ in the CM frame.

$$\nabla \equiv \left(\frac{\partial}{\partial \xi_1}, \frac{\partial}{\partial \xi_2}, \frac{\partial}{\partial \xi_3} \right). \quad (30)$$

The wavefunction ψ must satisfy one equation of motion for the satellite and one for the vertex. These equations come from the classical mass-shell conditions of equations 9 and 11. Using the first-quantized forms of D_0 and D_1 (equation 15), these mass-shell conditions give the following quantum equations of motion.

$$D_0\psi \equiv (\partial_{\mu}\partial^{\mu} - M^2)\psi = 0. \quad (31)$$

$$D_1\psi \equiv (\partial_{s\alpha}\partial_s^{\alpha} - m_1^2)\psi = 0, \quad (32)$$

The operator form of M comes from the quantum version of equation 10.

$$M = \pi^{\parallel} = i\frac{\partial}{\partial \xi^{\parallel}}. \quad (33)$$

The consistency condition of equation 14 translates to the following quantum form.

$$[D_0, D_1]\psi = 0. \quad (34)$$

As expected, this produces the same condition on m_1 as seen in the classical case (equation 17). If the functional form for m_1 is chosen to produce a confining effect on the satellite, the solutions of equations 31 and 32 will produce a discrete spectrum for the eigenvalues of π^{\parallel} and hence, M . This spectrum of values of M represent the possible values of effective mass for the composite particle. In a field theory, each eigenvalue of M will represent a different particle.

IV. SECOND QUANTIZATION OF COMPOSITE PARTICLES

For a second quantized theory of composite particles, it is important to notice that the vertex and the satellite are never individually free. This makes it possible to second quantize the whole composite without second quantizing the vertex or the satellite individually. When the whole composite is second quantized, the internal dynamics of vertex and satellite can be completely represented through quantum numbers for internal energy

(given by M) and angular momentum. These quantum numbers can be visualized as extensions of the set of intrinsic particle quantum numbers like spin.

The recipe for second quantization of composite particles is a generalization of the usual canonical second quantization procedure. We start by defining the following *universal current* as a generalization of conserved currents in theories of structureless particles.

$$j^{\mu\alpha} \equiv (1/4)\psi^\dagger \overset{\leftrightarrow}{\partial}{}^\mu \overset{\leftrightarrow}{\partial}{}^\alpha \psi, \quad (35)$$

where ψ^\dagger is the adjoint of ψ and $\overset{\leftrightarrow}{\partial}{}^\mu$ is defined by the following.

$$\psi^\dagger \overset{\leftrightarrow}{\partial}{}^\mu \psi \equiv \psi^\dagger (\overset{\rightarrow}{\partial}{}^\mu - \overset{\leftarrow}{\partial}{}^\mu) \psi \equiv \psi^\dagger (\partial^\mu \psi) - (\partial^\mu \psi^\dagger) \psi. \quad (36)$$

$\overset{\leftrightarrow}{\partial}{}^\alpha_s$ is defined similarly using the satellite relative coordinates. This yields the following conserved currents.

$$j^\mu \equiv \int j^{\mu\alpha} d^3\xi_\alpha, \quad j_s^\alpha \equiv \int j^{\mu\alpha} d^3x_\mu, \quad (37)$$

where terms like d^3x_μ represent four-vector hypersurface elements in x_μ space. The integrations are done over arbitrary infinite spacelike hypersurfaces. It is straightforward to prove the following conservation equations using the equations of motion[18].

$$\partial_{s\alpha} j_s^\alpha = 0, \quad (38)$$

and

$$\partial_\mu j^\mu = 0. \quad (39)$$

Both conserved currents lead to the same conserved charge. It is given by

$$\mathcal{Q} \equiv \int j^{\mu\alpha} d^3\xi_\alpha d^3x_\mu. \quad (40)$$

Due to the conservation equations 38 and 39 it can be seen that the two integrations over space-like hypersurfaces are independent of the choice of any specific hypersurface. Hence, for convenience, we choose the ξ_α hypersurfaces to be orthogonal to the total momentum P . This makes sure it is purely spatial in the CM frame.

So, we replace $d^3\xi_\alpha$ by $d^3\xi^\perp$ and $\overset{\leftrightarrow}{\partial}{}^\alpha_s$ by $\overset{\leftrightarrow}{\partial}{}^\parallel_s$ where

$$\overset{\leftrightarrow}{\partial}{}^\parallel_s \equiv -\hat{P}_\alpha \overset{\leftrightarrow}{\partial}{}^\alpha_s. \quad (41)$$

For the d^3x_μ integration we choose the purely spatial components \mathbf{x} in the laboratory frame. Hence, $\overset{\leftrightarrow}{\partial}{}^\mu$ can be replaced by $\overset{\leftrightarrow}{\partial}{}^0$. This gives the conserved charge to be

$$\mathcal{Q} \equiv (1/4) \int \psi^\dagger \overset{\leftrightarrow}{\partial}{}^0 \overset{\leftrightarrow}{\partial}{}^\parallel_s d^3\xi^\perp d^3\mathbf{x}. \quad (42)$$

This conserved charge suggests the following natural norm for the Hilbert space of ψ .

$$(\psi, \psi) \equiv 1/4 \int \psi^\dagger \overset{\leftrightarrow}{\partial}{}^0 \overset{\leftrightarrow}{\partial}{}^\parallel_s \psi d^3\xi^\perp d^3\mathbf{x}. \quad (43)$$

This leads to the following definition of the inner product.

$$(\phi, \psi) \equiv 1/4 \int \phi^\dagger \overset{\leftrightarrow}{\partial}{}^0 \overset{\leftrightarrow}{\partial}{}^\parallel_s \psi d^3\xi^\perp d^3\mathbf{x}. \quad (44)$$

The above inner product definition allows us to identify the following orthonormal basis for the set of solutions of the equations of motion.

$$\psi_{\mathbf{k}E} \equiv [k^0(2\pi)^3]^{-1/2} \Psi(E, \xi^\perp) \exp[-iE\xi^\parallel] \exp(ik \cdot x), \quad (45)$$

where k is the four-vector eigenvalue of the total momentum P , \mathbf{k} is its three-vector part and k^0 is its zeroth component. E is the CM energy of the satellite and hence, an eigenvalue of π^\parallel or M . Note that E , can be negative. This requires the usual explanation of an antiparticle being a particle going backward in time. So the physically measurable mass is still positive. For $\psi_{\mathbf{k}E}$ to be a solution of the equations of motion in the satellite sector, $\Psi(E, \xi^\perp)$ must satisfy the following eigenvalue equation.

$$H\Psi(E, \xi^\perp) = E\Psi(E, \xi^\perp), \quad (46)$$

where

$$H \equiv \sqrt{(\pi^\perp)^2 + m_1^2}, \quad (47)$$

It is to be noted that $\Psi(E, \xi^\perp)$ also depends on angular momentum quantum numbers due rotational symmetry. The labels for these quantum numbers are suppressed for brevity of notation. Also, the spectrum for E is expected to be discrete as m_1 produces a confining effect. For $\psi_{\mathbf{k}E}$ to be a solution of the whole particle equation of motion, the following must be satisfied.

$$k^0 = \sqrt{\mathbf{k}^2 + E^2}. \quad (48)$$

Ψ may be normalized in the usual fashion.

$$\int \Psi^\dagger(E', \xi^\perp) \Psi(E, \xi^\perp) d^3\xi^\perp = \delta_{E'E}, \quad (49)$$

where $\delta_{E'E}$ is the Kronecker delta and, once again, the angular momentum labels are suppressed and understood to be included in the corresponding energy label. Using these conditions, it can be verified that the $\psi_{\mathbf{k}E}$ are truly orthonormal.

$$(\psi_{\mathbf{k}'E'}, \psi_{\mathbf{k}E}) = \delta_{E'E} \delta(\mathbf{k}' - \mathbf{k}), \quad (50)$$

where $\delta(\mathbf{k}' - \mathbf{k})$ is the Dirac delta.

Now we are ready for second quantization. The standard prescription for canonical quantization will be used. However, it is critical to note that the satellite and the vertex are not second quantized individually. It is the

whole particle wavefunction ψ that is second quantized. The energy and angular momentum of the satellite are treated as extra degrees of freedom (quantum numbers) of the whole particle wavefunction.

First, a Lagrangian for the particle field is defined as follows.

$$\mathcal{L} = -1/2 \int \psi^\dagger \overleftrightarrow{\partial}_s^\dagger [\overleftarrow{\partial}_\mu \overrightarrow{\partial}^\mu + M^2] \psi d^3 \xi^\perp, \quad (51)$$

The momentum conjugate to ψ would then be

$$\phi \equiv \frac{\partial \mathcal{L}}{\partial (\partial_0 \psi)} = \partial_s^\parallel \partial^0 \psi^\dagger, \quad (52)$$

Then the second quantization condition can be written symbolically as the following equal-time commutator[21].

$$[\psi, \phi] = i\delta, \quad (53)$$

where the δ is a delta function over all degrees of freedom.

Now, ψ can be expanded in terms of the basis set of equation 45 as follows.

$$\begin{aligned} \psi = \int d^3 \mathbf{k} \sum_E [2k^0 (2\pi)^3]^{-1/2} \Psi(E, \xi^\perp) \exp[-iE\xi^\parallel] \cdot \\ \cdot [b(\mathbf{k}, E) \exp(i\mathbf{k} \cdot \mathbf{x}) + d^\dagger(\mathbf{k}, E) \exp(-i\mathbf{k} \cdot \mathbf{x})]. \end{aligned} \quad (54)$$

As in usual field theories, the b and d^\dagger coefficients are used to separate particle and antiparticle states. d^\dagger represents the hermitian adjoint of d in a field operator sense.

Then, the quantization condition of equation 53 reduces to the following (as before, the energy labels are understood to include angular momentum labels).

$$\begin{aligned} [b(\mathbf{k}, E), b^\dagger(\mathbf{k}', E')] &= [d(\mathbf{k}, E), d^\dagger(\mathbf{k}', E')] = \\ &= \delta^3(\mathbf{k} - \mathbf{k}') \delta_{E'E}, \end{aligned} \quad (55)$$

and all other commutators of b , b^\dagger , d and d^\dagger vanish. This allows the building of the usual Fock space with b^\dagger being the particle creation operator, b the particle annihilation operator, d^\dagger the antiparticle creation operator and d the antiparticle annihilation operator. The necessary vacuum state can be shown to be stable[18].

This is a field theory of a composite particle with a bosonic vertex and a bosonic satellite. It is possible to generalize this to composites with vertex and satellite each being either bosonic or fermionic[18, 19, 20]. It is also possible to have multiple satellites.

V. CONCLUSION

An unusual mechanism for mass generation is discussed here. It requires the postulation of an internal structure for particles. A satellite permanently attached to the bare particle is seen to generate mass dynamically. Hence, this satellite may be considered to be a first quantized equivalent of the Higgs boson. However, the satellite, being attached to the bare particle by confining forces, is not expected to be detected independently.

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